

The general multiplicative ARIMA/SARIMA framework can be written:

$$\phi_P(B^s)\phi_p(B)(1-B)^d(1-B^s)^D Z_t = \theta_q(B)\vartheta_Q(B^s)e_t \quad (1)$$

where B is backshift operator (i.e. Z(t)B=Z(t-1)) , and

$$\phi_P(B^s) = 1 - \phi_1 B^s - \phi_2 B^{2s} - \dots - \phi_p B^{ps} \quad (2)$$

$$\phi_p(B) = 1 - \phi_1 B - \phi_2 B^2 - \dots - \phi_p B^p \quad (3)$$

$$\theta_q(B) = 1 - \theta_1 B - \theta_2 B^2 - \dots - \theta_q B^q \quad (4)$$

$$\vartheta_Q(B^s) = 1 - \vartheta_1 B^s - \vartheta_2 B^{2s} - \dots - \vartheta_Q B^{Qs} \quad (5)$$

The general setting in equation (1) can also expressed as:

ARIMA(p,d,q)x(P,D,Q).

1- In the ARIMA(2,1,3) , we have p=2, d=1, q=3, s=0, then its mathematical structure can be shown as:

$$(1 - \phi_1 B - \phi_2 B^2)(1 - B)Z_t = (1 - \theta_1 B - \theta_2 B^2 - \theta_3 B^3)e_t$$

2- Similarly the structure of ARIMA(1,0,1)(0,1,1)<sub>12</sub> where (p=1,d=0,q=1; P=0, D=1,Q=1, S=12) is:

$$(1 - \phi_1 B)(1 - B^{12})Z_t = (1 - \theta_1 B)(1 - \vartheta_1 B^{12})e_t$$